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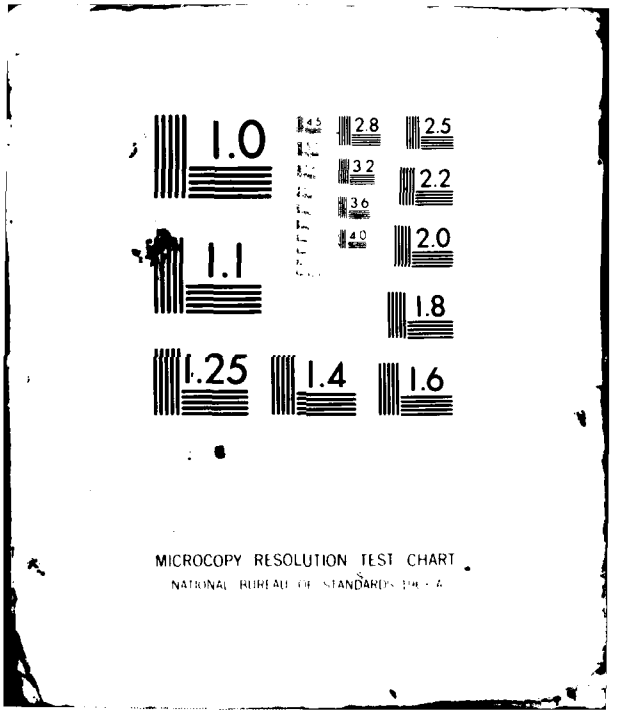
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Final Report

Grant AFOSR 77-3180

Principal Investigator: F.J. Samaniego



Sixteen research projects have been completed over the period October 1, 1976 to September 30, 1981 under the sponsorship of grant AFOSR 77-3180. These projects are summarized below. Sixteen technical reports (or their published counterparts) are submitted with this report under separate cover.

1. A characterization of convoluted geometric distributions. The distributions of geometric signals in discrete noise are shown to be characterized by a certain system of differential equations. An application to statistical inference is discussed.
2. A generalization of a theorem by Boswell and Patil. A characterization theorem for the negative binomial distribution due to Boswell and Patil is extended, yielding a characterization of convolutions of negative binomial distributions.
3. Maximum likelihood estimation for binomially distributed signals in discrete noise. The distribution of binomial signals in noise is characterized by systems of differential equations satisfied by their probability mass function. The result proves useful in maximum likelihood estimation, in that the binomial parameters of any convolution satisfying a certain motonicity condition is shown to be easily estimated by the method of maximum likelihood. Several examples are given to show the breadth of applicability of the results.
4. On the power of the $\chi^2$ goodness of fit test at signal plus noise alternatives. The likelihood ratio test is developed for examining certain goodness of fit questions and compared via simulation with small and moderate sample sizes to the $\chi^2$ test.

5. Two characterizations for Pascal signals in additive noise. This paper unifies and generalizes the results in 1 and 2 above.
6. Maximum likelihood estimation for a class of multinomial distributions arising in reliability. We note the difficulty of obtaining a closed form expression for the MLE in samples $Y_1,\ldots,Y_n$ where each Y is the sum of independent but nonidentically distributed Bernoulli variables. It is then demonstrated that the invariance property of MLE's can be used to advantage, yielding an estimator that is asymptotically equivalent to the MLE and is obtainable from the roots of a polynomial whose degree doesn't depend on the sample size. A reliability setting in which this estimation problem arises is discussed.
7. Pseudo maximum likelihood estimation: Theory and Applications. This paper presents a contribution to general statistical theory that is particularly useful in estimation problems dealing with signals in additive noise. The asymptotic behavior of solutions of a single likelihood equation in which all nuisance parameters are replaced by $O(1/\sqrt{n})$ estimators is given.
8. Bathtub and related failure rate characterizations. Sufficient conditions are given for a lifetime distribution to have a bathtub shaped failure rate. Examples are furnished which introduce new bathtub models and illustrate the use of the general results for existing models.
9. A unified treatment of modeling and inference for discrete signals in additive noise. In this paper, a classification theorem is given which simultaneously characterizes convolutions of Poisson, binomial and Pascal distributions. This paper provides the missing link in papers 3 and 5 above and a 1976 paper by the principal investigator on convoluted Poisson distributions. The connection is a subtle one, and establishing it requires a fairly exotic parameterization of a general two parameter discrete family.

10. Statistical inference for a new model in reliability. In paper 8 above, lifetime distributions with bathtub shaped failure rates were introduced. The current paper examines estimation and testing questions for a particular exponential family with the bathtub property. In particular, a goodness of fit test is developed for the exponential distribution against alternatives with a bathtub shaped failure rate.

11. Modeling and inference for positively dependent variables in dichotomous experiments. A discrete multivariate distribution is introduced which arises naturally in reliability experiments. Maximum likelihood estimation is shown to be intractable. An alternative estimator is obtained (in closed form) and shown to be asymptotically efficient.

12. Maximum likelihood estimation for a discrete multivariate shock model. The multivariate model studied in 11 above has $2^k-1$ dimensional parameter space, where $k$ is the dimension of the observed vector of zeros and ones. (The multivariate exponential distribution of Marshall and Olkin also has this feature). In this paper, a natural submodel is studied for which we obtain somewhat better results than for the general model. Moreover, the model has only $(k+1)$ parameters and the estimation problem is feasible even for moderate sample sizes.

13. Moment identies via integrated survival curves. The well known identity $EX = \int_0^\infty (1-F(x))dx$, where $P(X>0) = 1$, is generalized to a formula for computing arbitrary moments $EX^k$ as $k$ fold integrals. A general identity and a formula applicable only to discrete nonnegative variables are developed. Two applications are indicated.

14. Evaluating performance in continuous experiments with feedback to subjects. This paper extends the treatment of discrete experiments with feedback by Diaconis and Graham (Annals of Statistics, 1981) to continuous

guessing experiments. The optimal strategy is identified, and a test is proposed for the hypothesis that the subject's performance is due to chance. Skill scoring, a technique of evaluating performance which allows for the use of a suboptimal strategy by the subject, is developed in the context under study, and a test based on skill scoring is proposed.

15. Estimating the sib proportion in seed purity determinations. An estimation problem arising in plant breeding is examined from several perspectives. An efficient estimator is developed for the parameter of interest. Certain structural facts concerning the asymptotic variance of the estimator are established. We discuss the problem of optimal allocation of samples to the three populations from which samples are drawn. Sample size requirements for specified precision are discussed. The treatment of certain peculiarities of this estimation problem (eg, an asymptotic variance that is an unbounded function of model parameters) may be of independent interest.

16. Estimating a survival curve when new is better than used. Estimation within the nonparametric class of NBU distributions is treated. An estimator obtained from a transformation which constructs an NBU curve out of an arbitrary survival curve is shown to be consistent when the true survival curve has compact support. An important application of this result is to data subject to type I censoring. The rate of convergence of the estimator is shown to be optimal in such problems. The estimator is adapted to yield a consistent NBU sequence in the general problem. This paper is a revised and expanded version of a previously submitted report by the same title.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The report summarizes research progress under the grant in the following areas: (1) a characterization of convoluted geometric distributions; (2) a generalization of a theorem by Boswell and Patil; (3) maximum likelihood estimation for binomially distributed signals in discrete noise; (4) on the power of the $X^2$ goodness of fit test at signal plus noise alternatives; (5) two characterizations for Pascal signals in additive noise; (6) maximum likelihood estimation for a class of multinomial distributions arising in reliability; (CONTINUED)

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ITEM #20, CONTINUED: ~~(7)~~ pseudo maximum likelihood estimation: theory and applications; ~~(8)~~ bathtub and related failure rate characterizations; ~~(9)~~ a unified treatment of modeling and inference for discrete signals in additive noise; ~~(10)~~ statistical inference for a new model in reliability; ~~(11)~~ modeling and inference for positively dependent variables in dichotomous experiments; ~~(12)~~ maximum likelihood estimation for a discrete multivariate shock model; ~~(13)~~ moment identities via integrated survival curves; ~~(14)~~ evaluating performance in continuous experiments with feedback to subjects; ~~(15)~~ estimating the sib proportion in seed purity determinations; and ~~(16)~~ estimating a survival curve when new is better than used.